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DETERMINATION OF THE EFFECT OF WING FLEXIBILITY ON LATERAL MANEUVERABILITY AND A COMPARISON OF CALCULATED ROLLING EFFECTIVENESS WITH FLIGHT RESULTS

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DETERMINATION OF THE EFFECT OF WING FLEXIBILITY ON LATERAL MANEUVERABILITY AND A COMPARISON OF CALCULATED ROLLING EFFECTIVENESS WITH FLIGHT RESULTS

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SUMMARY

An analysis is made to show that, when account is taken of sideslip and wing flexibility, the calculated rolling maneuverability of an airplane is in good agreement with the results obtained from flight tests. The method used for taking into account the effect of wing flexibility avoids the complications of successive approximations but is nevertheless believed to be more nearly accurate than other methods based on semirigid-wing assumptions. The application of the method to a wing of tubular shell construction is considered, and the procedure is illustrated for a modern pursuit airplane.

INTRODUCTION

Flight results obtained from reference 1 and other sources indicate that the rolling effectiveness of airplanes is in many cases lower than that predicted from the theoretical method of reference 2, based on wind-tunnel aileron effectiveness. Reference 3, on the basis of a study of recent experimental data, has suggested the use of an empirical constant of 0.80 to account for the various factors contributing to the reduction of rolling effectiveness in flight. The present investigation was undertaken in order to determine a procedure that would enable designers to make a more nearly accurate prediction of the lateral maneuverability of airplanes. Inasmuch as the important factors affecting the rolling maneuverability appeared to be sideslip and wing flexibility, the present analysis is concerned principally with a careful determination of the influence of these factors and a comparison of the calculated with the flight results for rolling effectiveness when allowance is made for sideslip and wing flexibility.

Methods for predicting the effect of sideslip on lateral maneuverability are given in references 2, 4, and 5 but in the present investigation measured sideslip data were available and the data were utilized in the comparison of the calculated rolling effectiveness with the flight results.

A method for calculating the loss in lateral control due to wing twist is given in reference 6. The method presented therein, however, depends on an arbitrarily chosen shape for the spanwise twist distribution in conjunction with an empirically determined reference section. This procedure for obtaining the spanwise twist distribution, therefore, does not establish for any particular case the required equilibrium, at every section, between the aerodynamic torque in the rolling maneuver and the elastic force of the wing. For modern airplanes, moreover, on which the wings have cut-outs for the landing gear and armament that cause comparatively large variations in the spanwise torsional rigidity, it would be particularly unlikely that an accurate spanwise twist distribution could be obtained from an arbitrarily chosen shape of spanwise twist distribution and an empirically determined reference section.

In order to obtain greater accuracy in the calculations for the effect of wing flexibility on rolling maneuverability, a method is developed in the present investigation in which the spanwise twist distribution is computed on the basis of the actual wing elasticity rather than by the method of reference 6. The required equilibrium between the aerodynamic torque and the elastic force is established at every section with reasonable accuracy without the complications of the successive approximations ordinarily required to obtain this equilibrium. It is indicated that the method is applicable to modern wing designs having conventional ailerons.

SYMBOLS

L — rolling moment, assumed positive when rotation of right wing is downward; for contributing factors, see subscripts

f_1, f_2, and f_3 — factors denoting aspect-ratio correction applied to rolling moment computed on basis of two-dimensional flow

q — dynamic pressure, pounds per square foot $\left(\frac{1}{2}\rho V^2\right)$

p — rolling velocity, assumed positive when the right wing moves downward, radians per second

V — true airspeed, feet per second

a_o — slope of lift-coefficient curve per degree at infinite aspect ratio, $dc_l/d\alpha$

c_l — lift coefficient at a section, positive upward; for contributing factors, see subscripts

α — angle of attack at a section, degrees

V_i — indicated airspeed, miles per hour $\left(\frac{1}{1.467}\sqrt{\frac{2q}{\rho_o}}\right)$

ρ — air density

ρ_o — air density at sea level

c_w — wing chord at any section, feet

c_a — aileron chord at any section measured from hinge line to trailing edge, feet

y — coordinate measured along lateral axis of airplane, feet

y_1, y_2 — coordinates indicating, respectively, the fixed positions for the inboard and outboard edges of the aileron, feet

$\left(\frac{\partial \alpha}{\partial \delta_a}\right)_{c_n}$	rate of change of section angle of attack with aileron deflection for constant normal force at section; used with prime to indicate the value at the section for which wind-tunnel data were obtained
δ_a	aileron deflection, positive when the right aileron is deflected upward, degrees
$\Delta\delta_a$	total aileron deflection measured as the angle between the right and left ailerons, degrees
θ	wing twist, positive when the leading edge of right wing moves upward, degrees
b	wing span, feet
S	wing area
$\frac{dM}{dy}$	aerodynamic twisting moment per unit span taken about the aerodynamic center, positive for stalling moment, foot-pounds per foot
$\left(\frac{\partial c_m}{\partial \delta_a}\right)_{c_n}$	rate of change of pitching-moment coefficient per degree aileron deflection for constant normal force at section; symbol is primed to indicate the value at the section for which wind-tunnel data were obtained
e	distance from aerodynamic center to elastic center at a section, positive when aerodynamic center is ahead of elastic center, feet
C_L	over-all wing lift coefficient; for contributing factors, see subscripts
M	Mach number; in expression dM/dy, twisting moment
$\Delta(pb/2V)_s$	reduction in helix angle $pb/2V$ due to sideslip
$\Delta\delta_{a_s}$	total aileron deflection required to balance steady sideslip
$\left(\frac{pb}{2V}\right)_{fl}$	value of the helix angle $pb/2V$ measured in a roll from flight data

$\Delta\delta_{a_{fl}}$ total aileron deflection measured in a roll from flight data

y' coordinate indicating fixed spanwise position, feet

T torsional moment acting outboard of a section

$T_{y'}$ total aerodynamic twisting moment acting on wing outboard of a section, foot-pounds, $\left(\int_{y'}^{b/2} \frac{dM}{dy}\, dy\right)$

M' concentrated torque applied at section near wing tip, foot-pounds

$C_{T.R.}$ coefficient of torsional rigidity along span, which is equal to $\frac{M'}{d\theta/dy}$, where $d\theta/dy$ is slope of deformation curve resulting from concentrated torque M'

G modulus of elasticity in shear

A_m area enclosed by line midway between the inner and outer boundaries of thin-walled section of wing

t wall thickness of wing section

s perimeter measured by line midway between inner and outer boundaries of thin-walled section of wing

K torsion factor equal to $4A_m^2 \Big/ \oint_c \frac{ds}{t}$ for thin-walled tube in which skin has not buckled

$B = q\delta_a\left[(\partial c_m/\partial\delta_a)_{c_n}\right]'$

Subscripts:

damp used to denote contribution of aerodynamic damping to aerodynamic characteristics of airplane

aileron used to denote contribution of aileron deflection to aerodynamic characteristics of airplane twist used to denote contribution of wing twisting to aerodynamic characteristics of airplane

ANALYSIS

The assumption is made that, during the steady phase of a pure roll following the application of the ailerons, the rolling moments of an airplane due to the aerodynamic damping, the aileron deflection, and the wing twist are in equilibrium. Thus,

$$L_{damp} + L_{aileron} + L_{twist} = 0 \qquad (1)$$

The changes in geometric incidence at any section y, which result from the damping, the aileron deflection, and the wing twist are, respectively, $\frac{py}{V}$, $\left(\frac{\partial \alpha}{\partial \delta_a}\right)_{c_n} \delta_a$, and θ. From the lifting-line theory (reference 7), therefore, for a symmetrical wing-aileron arrangement, equation (1) becomes

$$57.3 f_1 \frac{qp}{V} \int_0^{b/2} a_o c_w y^2 \, dy$$

$$= f_2 q \int_{y_1}^{y_2} a_o \left(\frac{\partial \alpha}{\partial \delta_a}\right)_{c_n} \delta_a c_w y \, dy - f_3 q \int_0^{b/2} a_o \theta c_w y \, dy \qquad (2)$$

where the normally insignificant rolling-moment contribution due to the drag is neglected, and where p is taken to be positive when the right wing moves downward.

In equation (2), f_1, f_2, and f_3 are the aspect-ratio corrections for the appropriate geometric angle-of-attack distribution and plan form and the aerodynamic parameters a_o and $(\partial \alpha / \partial \delta_a)_{c_n}$ refer to values appropriate to a Mach number and lift coefficient for the altitude and dynamic pressure q under consideration. Reference 7 shows that the aspect-ratio correction for an elliptical plan form is independent of the spanwise distribution of geometric angle of attack. Calculations indicate, also, that for a wing having conventional ailerons and a plan form that approximates the elliptical (such as wings having taper ratios of about 1.7:1 to 4:1), differences in the values of f_1, f_2, and f_3 will usually be negligible. For these cases, therefore, it appears justifiable to eliminate f_1, f_2, and f_3 from equation (2). (For special cases, where the plan form departs from the elliptical to a greater extent than in the taper ratios mentioned, the rolling moments in equation (1) may be obtained by the method and data given in reference 2, in which the antisymmetrical change in geometric angle of attack due to wing twist θ, which is to be determined herein, can be treated in the same manner as that indicated for the change in angle of attack due to aileron deflection $(\partial\alpha/\partial\delta_a)_{c_n}$.)

The distributions of spanwise twist θ for use in equation (2) may be obtained from a consideration of the aerodynamic torque and the elastic forces acting on the wing.

During the rolling maneuver the lift force at any section consists of the component contributed by the aileron deflection, which acts at the center of pressure, and the components due to the aerodynamic damping and wing twisting, which act at the aerodynamic center of the section. This resultant chordwise lift distribution gives a twisting moment at each section having the value

$$dM = \delta_a\left(\frac{\partial c_m}{\partial\delta_a}\right)_{c_n} qc_w^{\,2}\,dy$$
$$+ \left[c_{l_{aileron}} + \left(c_{l_{damp}} + c_{l_{twist}}\right)\right]\frac{eqc_w^{\,2}}{c_w}\,dy \qquad (3)$$

In equation (3), c_m is taken about the aerodynamic center of the section; the term in brackets is the resultant lift coefficient for the components due to the aileron deflection, aerodynamic damping, and wing twisting; and e/c_w is the distance as a fraction of the chord from the aerodynamic center to the elastic center, reckoned as positive to the rear. In this equation, the first term on the right-hand side represents the total twisting moment of the section if the elastic axis coincides with the aerodynamic center and the second term gives the additional twisting moment due to the displacement of the elastic axis from the aerodynamic center. A consideration of the additional twisting moment contributed by the displacement of the elastic center from the aerodynamic center shows that the twisting moment will usually be small for conventional wing-aileron systems in which the ailerons have a span of about 40 to 50 percent of the wing span and extend to the spanwise position of about 90 to 100 percent of the wing semispan. This low value for the additional twisting moment follows from the fact that the three-dimensional lift distributions due to the aileron deflection, aerodynamic damping, and wing twisting tend to have similar shapes because the preponderance of the geometric angle-of-attack distribution due to each of these components is in the outboard region of the wing; consequently, because of the equilibrium of the rolling moment and the similar shapes for the lift distribution of the components, the magnitude of $C_{L_{damp}} + C_{L_{twist}}$, when each half of the wing is considered separately, will generally be opposite in sign and of the same order as the magnitude of $C_{L_{aileron}}$. The factor e/c_w is also small for usual wing sections because the elastic center is in the vicinity of the aerodynamic center. The additional twisting moment in the case of conventional wing-aileron systems, therefore, will normally represent the product of two small terms; hence, in most cases, for practical limits of accuracy, the last term in equation (3) may be neglected as a second-order quantity.

In order to estimate the magnitude of the effect on the rolling maneuverability of the additional twisting moment due to the displacement of the elastic axis from the axis of aerodynamic centers, computations were made for a typical wing-aileron system having a 40-percent aileron span extending to the wing tip in which the elastic axis was assumed to be at a constant distance of 10 percent of the chord length behind the axis of aerodynamic centers. The computations utilized experimental data (furnished by the Army Air Forces), which were obtained from torsional-rigidity tests for the P-47B wing. On the basis of these calculations it is estimated that the effect of the 10-percent displacement of the elastic axis behind the axis of aerodynamic centers would be to increase the rolling effectiveness by an amount of the order of 1 percent or less for the complete range of speeds up to aileron reversal. Inasmuch as the elastic axis in modern wing designs is usually located within 15 percent of the chord length from the aerodynamic center, the conclusion regarding the negligible effect of the additional twisting-moment term in equation (3) appears to be justified.

The subsequent analysis will consider a wing of tubular shell construction. The twist of a section at a length y' from the wing center line may be expressed as

$$\theta_{y'} = \int_0^{y'} \frac{d\theta}{dy}\, dy \tag{4}$$

It is shown in references 8 and 9 for the case of tubes having closed sections, such as wings in which the wall or skin is thin in comparison with the other dimensions, that the angular twist at any section of infinitesimal width dy can be expressed in the form

$$\frac{d\theta}{dy} = \frac{T}{KG} \tag{5}$$

where T is the torsional moment acting outboard of the section, G is the modulus of elasticity in shear at the section, and K is a factor depending on the dimensions of the section and, as long as buckling of the skin does not occur,

$$K = \frac{4A_m^{\,2}}{\int_c ds/t}$$

If a concentrated torque is applied at a section near the wing tip, T in equation (5) is constant along the span and is equal to M' by definition; consequently, equation (5) may be written

$$KG = \frac{M'}{d\theta/dy}$$

$$= C_{T.R.}$$

by definition of $C_{T.R.}$.

The factors K and G depend only on the modulus of elasticity in shear and on the dimensions of the section and can therefore be considered invariant for equivalent loads on the wing as regards T obtained by either a concentrated or a distributed torque; consequently, the equality of the product KG to $C_{T.R.}$ is similarly valid for the case where T varies along the span, or, from equation (5),

$$\frac{d\theta}{dy} = \frac{T}{C_{T.R.}}$$

If this value for $d\theta/dy$ is substituted into equation (4), the twist at the spanwise position y' becomes

$$\theta_{y'} = \int_0^{y'} \frac{T}{C_{T.R.}} dy \qquad (6)$$

In practice, the variation of $C_{T.R.}$ along the span is usually determined by applying a pure twisting couple M' at a section near the wing tip and obtaining the slope of the deformation curve $d\theta/dy$ from measured values of the angular twist at various points along the span. The foregoing procedure for determining the spanwise distribution of twist θ in a rolling maneuver is illustrated for the case of a modern pursuit airplane in table I(c) and in figures 2, 3, and 4.

As a result of the foregoing analysis, for the case of conventional wing-aileron systems having approximately elliptical plan forms of taper ratios from about 1.7:1 to 4:1, equation (2) may be written

$$57.3\frac{p}{V}\int_0^{b/2} a_o c_w y^2 \, dy$$

$$= \int_{y_1}^{y_2} a_o \left(\frac{\partial \alpha}{\partial \delta_a}\right)_{c_n} \delta_a c_w y \, dy - \int_0^{b/2} a_o \theta c_w y \, dy \qquad (7)$$

where θ is determined from equation (6), in which the value of T at any spanwise position y is:

$$T = \int_{y}^{b/2} \frac{dM}{dy}\, dy = q \int_{y}^{b/2} \delta_a \left(\frac{\partial c_m}{\partial \delta_a}\right)_{c_n} c_w{}^2\, dy \qquad (8)$$

Equations (7) and (8) are also applicable to the case of a symmetrical wing-aileron plan form with differentially operated ailerons provided that the average aileron deflection is used for δ_a. If $\left(\partial c_m/\partial \delta_a\right)_{c_n}$ is obtained from low-speed wind-tunnel results, the value of this parameter should be multiplied by a compressibility correction factor, such as $1/\sqrt{1 - M^2}$, when used in equation (8). On the right-hand side of equation (7) the first term represents the part of the rolling effectiveness contributed by the rigid wing and the second term represents the reduction of rolling effectiveness due to wing flexibility. The speed V is contained in equation (7) in the expression for θ, since θ is expressed in equation (6) as a function of T, and T is expressed in equation (8) as a function of q (or $\frac{1}{2}\rho V^2$). The aileron reversal speed can be obtained from equation (7) by plotting p or $pb/2V$ against V and noting the speed corresponding to the intersection of the curve with the horizontal axis. If $\left(\partial \alpha/\partial \delta_a\right)_{c_n}$ and $\left(\partial c_m/\partial \delta_a\right)_{c_n}$ can be expressed analytically with reasonable accuracy as functions of V, the aileron reversal speed can be obtained by setting the left member of equation (7) equal to zero and solving the equation for V through θ as previously explained.

RESULTS AND DISCUSSION

Calculated Results

Calculations were made by the foregoing method for the rolling effectiveness of a modern pursuit airplane at various speeds. The details of the computations are given in order to illustrate an application of the method.

The calculations were made for the P-47C-1-RE airplane for a range of indicated airspeeds from 150 miles per hour to the aileron reversal speed at an altitude of approximately 4000 feet. The two-dimensional slope of the lift-coefficient curve a_o was assumed constant along the span and was therefore eliminated from equation (7). The dimensions of the wing-aileron system were obtained from drawings supplied by the Republic Aviation Corporation and are given in figure 1 and table I. The values for the aerodynamic parameters $\left(\frac{\partial \alpha}{\partial \delta_a}\right)_{c_n}$ and $\left(\frac{\partial c_m}{\partial \delta_a}\right)_{c_n}$ in equations (7) and (8), respectively, were based on two-dimensional test results obtained from unpublished tests made in the NACA 8-foot high-speed tunnel for a section at the midaileron span of the P-47C-1-RE airplane. Because the ratio of aileron chord to wing chord varied significantly along the span, the test results for the midaileron section were extrapolated on the basis of the theoretical curves of figure 1 of reference 10 in order to obtain the corresponding values at the other aileron sections; that is, it was assumed that the ratio of the actual aileron effectiveness at any section to the theoretical value was the same as the corresponding ratio deduced for the section tested in the wind tunnel. Thus,

$$\left(\frac{\partial \alpha}{\partial \delta_a}\right)_{c_n} = \frac{\left[\left(\frac{\partial \alpha}{\partial \delta_a}\right)_{c_n}\right]_{\text{theor}}}{\left[\left(\frac{\partial \alpha}{\partial \delta_a}\right)_{c_n}\right]'_{\text{theor}}} \left[\left(\frac{\partial \alpha}{\partial \delta_a}\right)_{c_n}\right]' \tag{9}$$

where the primed symbols refer to the values as obtained for the section tested in the wind tunnel. A corresponding relationship was also assumed for $\left(\frac{\partial c_m}{\partial \delta_a}\right)_{c_n}$. The variation with V_i of the parameters $\left[\left(\frac{\partial \alpha}{\partial \delta_a}\right)_{c_n}\right]'$ and $\left[\left(\frac{\partial c_m}{\partial \delta_a}\right)_{c_n}\right]'$ is shown in figure 2. The values given in the figure are based on the unpublished data from the 8-foot high-speed tunnel for an aileron deflection of ±4° at a wing lift coefficient and Mach number appropriate with reasonable accuracy to the P-47C-1-RE airplane at an altitude of approximately 4000 feet.

The torsional rigidity of the wing was obtained from experimental data furnished by the Army Air Forces, Materiel Center, Wright Field, Ohio for a prototype P-47B airplane. The P-47C-1-RE airplane wing structure is the same as that for the P-47B, although the sharp-nose Frise ailerons of the P-47B were modified for the P-47C-1-RE by introducing a blunter nose. The tests at Wright Field were made by applying a pure twisting couple at a section near the wing tip and measuring the angular twist at various stations along the span. The variations along the wing semispan of the twist θ per unit M' and of the torsional-rigidity coefficient $C_{T.R.}$ as obtained by the foregoing tests are shown in figure 3. The spanwise variations of the aerodynamic twisting moment due to the rolling maneuver dM/dy and the resulting total twisting moment outboard of any section T were calculated by means of equation (8). In the computations the effect of the displacement of the elastic center from the aerodynamic center on the aerodynamic torque due to the rolling maneuver was neglected because data obtained from the Republic Aviation Corporation indicated that the elastic axis for the P-47C-1-RE wing was of the order of $5\frac{1}{2}$ percent of the chord length back of the quarter-chord point. The spanwise twist distribution during the maneuver was computed from equation (6) by obtaining the value of $T/C_{T.R.}$ at several stations along the span and plotting these values as a function of the spanwise position y. The twist at any section is then equal to the area of the resultant curve measured from the center of the wing span to the desired station. The distributions of dM/dy, T, and θ in terms of the aileron deflection, dynamic pressure, and pitching-moment-coefficient derivative at the test section $\left[\left(\frac{\partial c_m}{\partial \delta_a}\right)_{c_n}\right]'$ are shown in figure 4.

The detailed steps and the numerical results obtained in the evaluation of the three members of equation (7) per unit aileron deflection are shown in table I. For convenience in making the summation indicated at the end of this table, the respective formulas for the three terms are referred to as I(a), I(b), and I(c). The specific computations for each term are given separately in the (a), (b), and (c) parts of the table. It should be noted at this point that the unit aileron deflection referred to is for one aileron and consequently the graphical integration given in the table is divided by 2 in order to present the results in terms of the total aileron deflection $\Delta\delta_a$.

On the basis of the foregoing data,

$$\frac{pb/2V}{\Delta\delta_a} = 0.00819\left[\left(\frac{\partial\alpha}{\partial\delta_a}\right)_{c_n}\right]' - 0.000237q\left[\left(\frac{\partial c_m}{\partial\delta_a}\right)_{c_n}\right]' \qquad (10)$$

where $\frac{pb/2V}{\Delta\delta_a}$ is the value of the helix angle per degree total aileron deflection measured as the angle between the right and left ailerons. Values for $\left[\left(\frac{\partial\alpha}{\partial\delta_a}\right)_{c_n}\right]'$ and $\left[\left(\frac{\partial c_m}{\partial\delta_a}\right)_{c_n}\right]'$ for use in equation (10) were obtained from figure 2 at the V_i corresponding to the dynamic pressure q.

The results of the calculations are presented in figure 5. Figure 5(a) gives the variation with V_i of the effective helix angle $pb/2V$ per degree total aileron deflection both for an assumed rigid wing and for the actual flexible wing in a pure roll at an altitude of approximately 4000 feet. The figure shows that, at V_i = 400 miles per hour, the effect of wing flexibility is to reduce $\frac{pb/2V}{\Delta\delta_a}$ from 0.00343 to 0.00239, and that aileron reversal occurs at V_i = 545 miles per hour. Figure 5(b) summarizes the calculated results from figure 5(a) and gives the variation with V_i of the ratio of $\frac{pb/2V}{\Delta\delta_a}$ for the flexible wing to the value for the assumed rigid wing. This figure shows that at V_i = 400 miles per hour, the ailerons for the P-47C-1-RE airplane are only 69 percent as effective in the actual flexible wing as in an assumed rigid one. These quantitative results are based on data for a total aileron deflection of 8°. Because of the variation of compressibility effects with aileron deflection for Frise ailerons the quantitative results may be noticeably different for very small deflections.

Comparison of Calculated and Flight Results

Figure 6 presents a comparison of the calculated rolling effectiveness with flight results for the P-47C-1-RE airplane for a range of V_i from 150 to 405 miles per hour at an altitude of approximately 4000 feet. The calculated results show the rolling effectiveness for the assumed rigid wing and also the rolling effectiveness when allowance is made for the wing twist and sideslip which accompanied the actual rolling maneuver. The flight data shown in figure 6 are based on unpublished results from tests conducted by the NACA on the P-47C-1-RE airplane. In these tests the angular deflections of the ailerons represent values measured at the inboard edge of the aileron span. The measured aileron deflections thus eliminate the factor of stretch in the aileron control system but the assumption is made that the aileron deflection at the inboard edge of the aileron span is representative of the deflections over the entire aileron span.

In figure 6, curve A gives the calculated value for $\frac{pb/2V}{\Delta\delta_a}$ for the assumed rigid wing in pure rolling. Curve B presents the results of curve A corrected for the sideslip and wing flexibility. The magnitudes of the corrections due to sideslip as represented by curve C were obtained by taking the measured values of the sideslip at the time of maximum rolling velocity and then employing flight data based on the P-47B airplane for the aileron deflection required to balance the measured magnitude of steady sideslip. As the rolling criterion $pb/2V$ is directly proportional to δ_a, the ratio of the aileron deflection required to balance the sideslip to the deflection measured in the roll is equal to the corresponding ratio of the loss of $pb/2V$ caused by the sideslip to the sum of the measured $pb/2V$ and the magnitude of the reduction in $pb/2V$ contributed by the sideslip. This relationship may be expressed in the form

$$\frac{\Delta\left(\frac{pb}{2V}\right)_s}{\left(\frac{pb}{2V}\right)_{fl} + \Delta\left(\frac{pb}{2V}\right)_s} = \frac{\Delta\delta_{a_s}}{\Delta\delta_{a_{fl}}}$$

or by the equivalent formula

$$\Delta\left(\frac{pb}{2V}\right)_s = \frac{\Delta\delta_{a_s}}{\Delta\delta_{a_{fl}} - \Delta\delta_{a_s}}\left(\frac{pb}{2V}\right)_{fl}$$

where $\Delta\left(\frac{pb}{2V}\right)_s$ refers to the loss in pb/2V due to sideslip, $\Delta\delta_{a_s}$ is the total aileron deflection required to balance the sideslip, and the subscript fl is used to indicate the measured values obtained in flight. The reduction in $\frac{pb/2V}{\Delta\delta_a}$ due to wing flexibility given in curve D of figure 6 represents the difference in rolling effectiveness between the rigid and flexible wing as determined from figure 5(a). The flight results in figure 6 (designated by circles) represent the average value of $\frac{pb/2V}{\Delta\delta_a}$ for right and left rolls. The flight values were obtained for a total aileron deflection of 8° by plotting the measured values of pb/2V against $\Delta\delta_a$ for each of the indicated airspeeds and using the faired values of pb/2V at $\Delta\delta_a = 8^\circ$.

The comparison in figure 6 of the calculated results with the flight results indicates good agreement when the calculated values for $\frac{pb/2V}{\Delta\delta_a}$ are corrected for wing flexibility and sideslip. The greater values of rolling effectiveness in flight than the calculated values, at speeds above approximately V_i = 350 miles per hour, may be explained to some extent by the fact that the flight results are based on aileron deflections measured at the inboard edge of the aileron, whereas the crank for the P-47C-1-RE aileron control system is located at the center of the aileron span; consequently, the effective aileron deflection along the span is likely to be somewhat greater than the value measured at the inboard edge because of the twisting of the torque tube.

On the basis of the foregoing comparison, it appears that, when stretch in the control system is neglected, the usual discrepancy which has been found between wind-tunnel and flight aileron effectiveness can be fully accounted for by the sideslip and wing twist that accompanies the roll.

In figure 6, as is to be expected, the reduction in $\frac{pb/2V}{\Delta\delta_a}$ due to sideslip varies approximately inversely as the square of the speed; whereas the loss due to wing flexibility increases approximately as the square of the speed. On this basis the trend is for the flight results for a certain range of comparatively low speeds to show little or no reduction in aileron effectiveness with increasing speed because the reduction in $pb/2V$ due to the wing twist is being compensated for by the increase in $pb/2V$ due to the smaller sideslip at the higher speed.

CONCLUSIONS

1. The calculated results of the present analysis indicate that the ailerons of the P-47C-1-RE airplane when deflected $\pm 4^\circ$ at 400 miles per hour indicated airspeed at approximately 4000 feet altitude are only 69 percent as effective in the actual flexible wing as in an assumed rigid wing, and aileron reversal occurs at 545 miles per hour indicated airspeed.

2. The comparison of the calculated rolling effectiveness based on wind-tunnel data for the aerodynamic parameters of the wing-aileron system indicates good agreement with available flight results when allowance is made for the sideslip and wing twist which accompanied the roll.

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TABLE I.- COMPUTATIONS FOR $\frac{pb}{2V}/\Delta\delta_a$; P-47C-1-RE AIRPLANE

Distance from center line, y (ft)	I(a) $57.3\frac{p}{V}\int_0^{b/2} c_w y^2\,dy$			I(b) $\int_{y_1}^{y_2}\left(\frac{\partial\alpha}{\partial\delta_a}\right)_{cn} c_w y\,dy$				
	c_w (ft)	$c_w y$ (ft^2)	$c_w y^2$ (ft^3)	Aileron chord, from hinge line, c_a (ft)	c_a/c_w	$\left[\left(\frac{\partial\alpha}{\partial\delta_a}\right)_{cn}\right]_{theor}$ Reference 10, fig. 1	$\frac{(\partial\alpha/\partial\delta_a)_{cn}}{\left[(\partial\alpha/\partial\delta_a)_{cn}\right]'} = \frac{\left[\left(\frac{\partial\alpha}{\partial\delta_a}\right)_{cn}\right]_{theor}}{0.55}$ (from equation (9))	$\frac{(\partial\alpha/\partial\delta_a)_{cn}}{\left[(\partial\alpha/\partial\delta_a)_{cn}\right]'} c_w y$
0	9.04	0	0					
2.42	9.00	21.76	52.59					
4.33	8.83	38.28	165.9					
5.33	8.71	46.46	247.8					
8.67	8.19	70.97	615.2					
11.00	7.70	84.74	932.3	1.10	0.143	0.465	0.845	71.64
13.50	7.03	94.87	1281	1.32	.188	.537	.976	92.63
15.17	6.47	98.12	1488	1.29	.200	.550	1.000	98.12
16.00	6.06	96.99	1552	1.27	.209	.560	1.018	98.74
17.67	5.20	91.81	1622	1.03	.198	.545	.991	90.98
19.34	3.64	70.43	1362	.40	.110	.415	.755	53.14
19.92	2.75	54.78	1091					
20.39	0	0	0					

By graphical integration,

$$\int_0^{b/2} c_w y^2\,dy = 16{,}300\ \text{ft}^4$$

$$I(a) = 57.3\frac{p}{V}\int_0^{b/2} c_w y^2\,dy = \frac{57.3}{20.3}\,\frac{pb}{2V}\times 16{,}300 = 45{,}500\,\frac{pb}{2V}\ \text{ft}^3$$

By graphical integration,

$$\int_{y_1}^{y_2}\frac{\left(\frac{\partial\alpha}{\partial\delta_a}\right)_{cn}}{\left[\left(\frac{\partial\alpha}{\partial\delta_a}\right)_{cn}\right]'} c_w y\,dy = 745\ \text{ft}^3$$

$$I(b) = \int_{y_1}^{y_2}\left(\frac{\partial\alpha}{\partial\delta_a}\right)_{cn} c_w y\,dy = 745\left[\left(\frac{\partial\alpha}{\partial\delta_a}\right)_{cn}\right]'\ \text{ft}^3$$

$$\frac{I(b)}{\Delta\delta_a} = \frac{745}{2}\left[\left(\frac{\partial\alpha}{\partial\delta_a}\right)_{cn}\right]'\ \text{ft}^3$$

Distance from center line, y (ft)	I(c) $\int_0^{b/2}\theta c_w y\,dy$								
	$\left[\left(\frac{\partial c_m}{\partial\delta_a}\right)_{cn}\right]_{theor}$ Reference 10, fig. 1	$\frac{(\partial c_m/\partial\delta_a)_{cn}}{\left[(\partial c_m/\partial\delta_a)_{cn}\right]'} = \frac{\left[\left(\frac{\partial c_m}{\partial\delta_a}\right)_{cn}\right]_{theor}}{0.0112}$	c_w^2 (ft^2)	$\frac{1}{B}\frac{dM}{dy} = \frac{(\partial c_m/\partial\delta_a)_{cn}}{\left[(\partial c_m/\partial\delta_a)_{cn}\right]'} c_w^2$ (ft^2)	$\frac{T}{B} = \int_y^{b/2}\frac{1}{B}\frac{dM}{dy}dy$ (ft^3)	$C_{T.R.}$ $\left(\frac{\text{lb-ft}^2}{\text{deg}}\right)$ from fig. 3	$\frac{T/B}{C_{T.R.}}$ $\left(\frac{\text{ft-deg}}{\text{lb}}\right)$	$\frac{\theta}{B} = \int_0^y \frac{T/B}{C_{T.R.}}dy$ $\left(\frac{\text{ft}^2\ \text{deg}}{\text{lb}}\right)$	$\frac{\theta c_w y}{B}$ $\left(\frac{\text{ft}^4\text{deg}}{\text{lb}}\right)$
0			81.63		323.2	∞	0		0
2.42			81.04		323.2	∞	0		0
4.33			78.00		323.2	$\to\infty$	0		0
5.33			75.86		323.2	1,900,000	0.00017	0.000080	0.0037
8.67			67.04		323.2	162,000	.001995	.002024	.2146
11.00	0.0104	0.929	59.32	55.09	323.2	83,500	.003871	.009844	.8342
13.50	.0111	.991	49.37	48.93	195.1	48,600	.004014	.02048	1.943
15.17	.0112	1.000	41.84	41.84	120.1	38,900	.003087	.02636	2.587
16.00	.0112	1.000	36.72	36.72	87.3	36,100	.002410	.02859	2.775
17.67	.0112	1.000	27.00	27.00	32.8	34,000	.000966	.03120	2.865
19.34	.0096	.857	13.26	11.37	0	31,200	0	.03190	2.247
19.92			7.56		0	--------	0	.03190	1.748
20.39			0		0	--------	0	.03190	0

By graphical integration,

$$\int_0^{b/2}\frac{\theta c_w y}{B}\,dy = 21.60\ \text{ft}^3$$

$$I(c) = \int_0^{b/2}\theta c_w y\,dy = 21.60B\ \text{ft}^2$$

$$\frac{I(c)}{\Delta\delta_a} = \frac{21.60}{2}q\left[(\partial c_m/\partial\delta_a)_{cn}\right]'\ \text{ft}^3$$

By use of equation (7) with δ_a given in terms of the total aileron deflection,

$$\frac{I(a)}{\Delta\delta_a} = \frac{I(b)}{\Delta\delta_a} - \frac{I(c)}{\Delta\delta_a}$$

$$\frac{45500}{\Delta\delta_a}\frac{pb}{V} = \frac{745}{2}\left[\left(\frac{\partial\alpha}{\partial\delta_a}\right)_{cn}\right]' - \frac{21.60}{2}q\left[\left(\frac{\partial c_m}{\partial\delta_a}\right)_{cn}\right]'$$

$$\frac{pb/2V}{\Delta\delta_a} = 0.00819\left[\left(\frac{\partial\alpha}{\partial\delta_a}\right)_{cn}\right]' - 0.000237q\left[\left(\frac{\partial c_m}{\partial\delta_a}\right)_{cn}\right]'$$

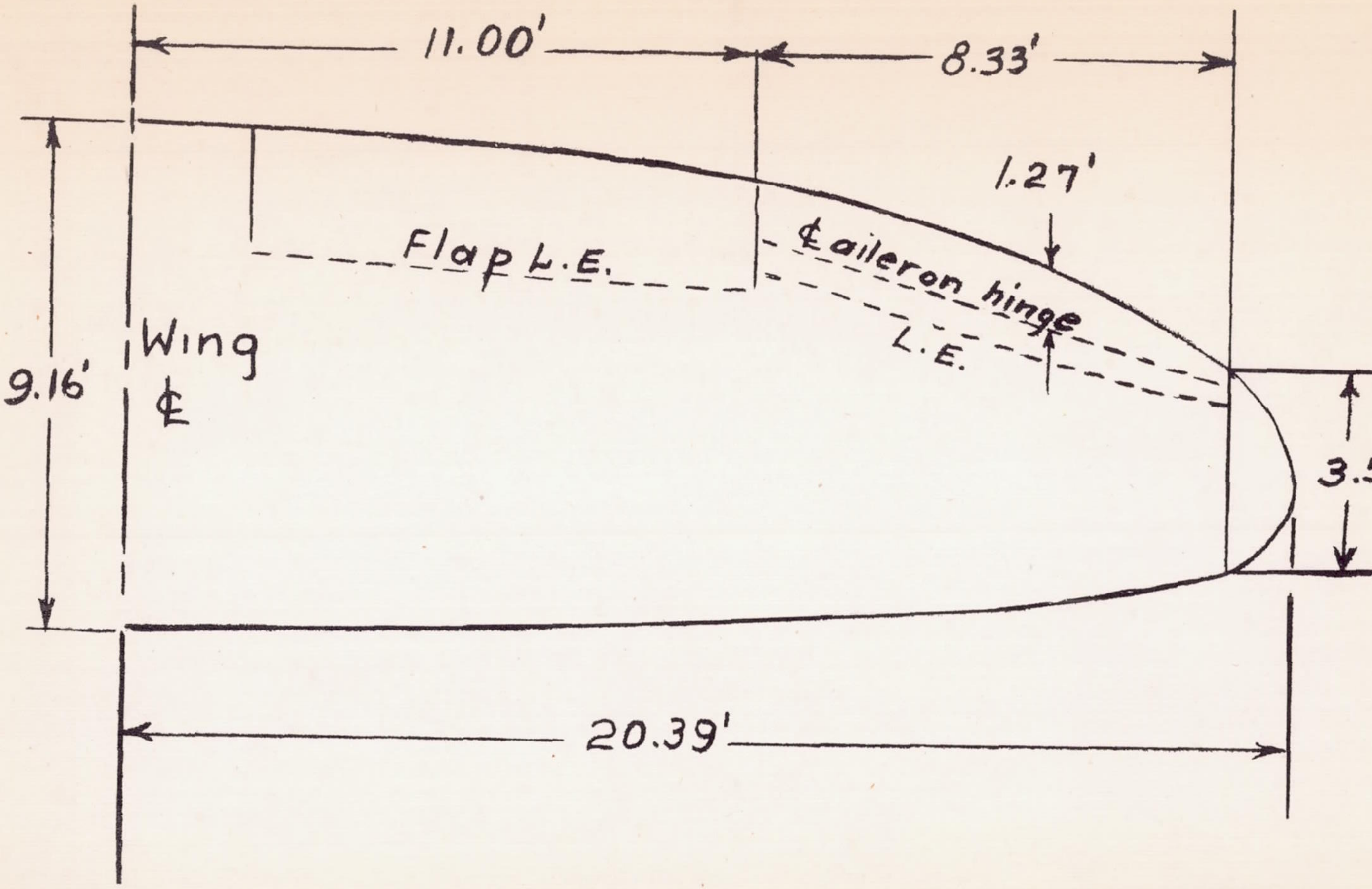

Figure 1.– Plan form of wing semispan showing wing and aileron dimensions. P-47C-1-RE airplane; wing area, 300 square feet; aileron area, 25.7 square feet.

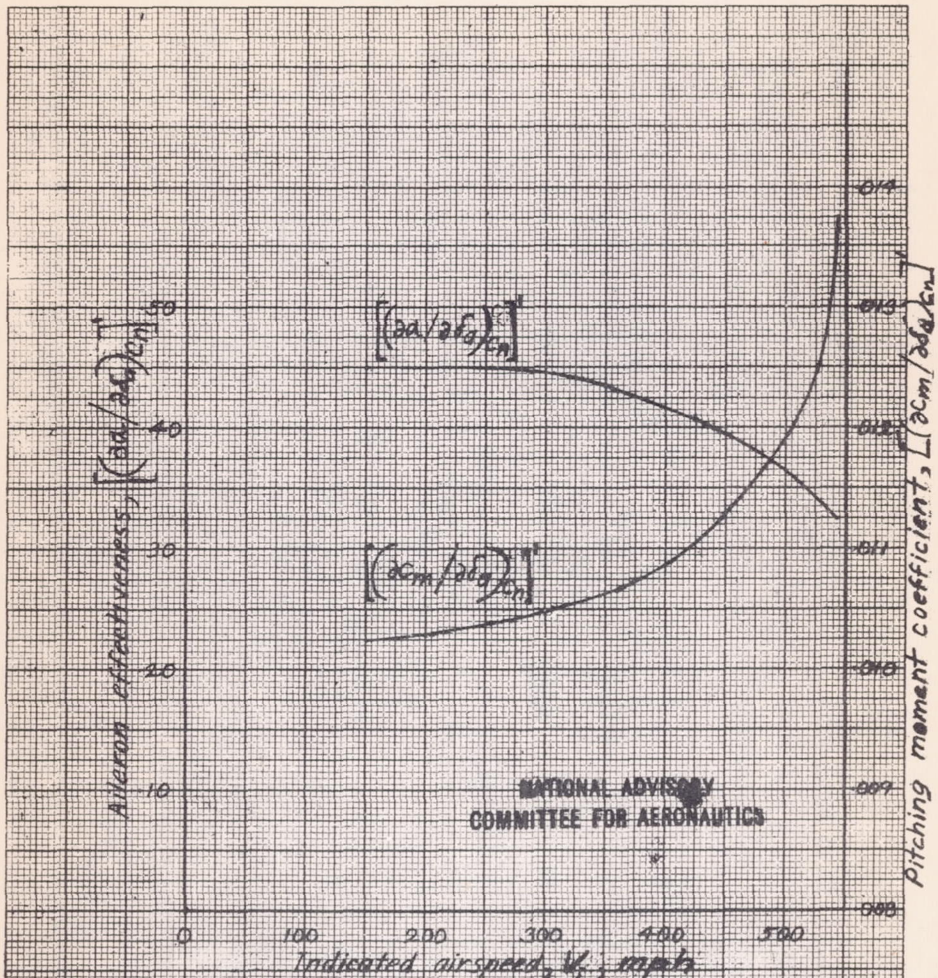

Figure 2.— Variation with indicated airspeed of parameters $\left[(\partial a/\partial \delta_a)_{c_n}\right]'$ and $\left[(\partial c_m/\partial \delta_a)_{c_n}\right]'$ for section at midaileron span from wind-tunnel results. Modified Frise aileron; $\delta_a = \pm 4^{\circ}$, lift coefficient and Mach number appropriate for P-47C-1-RE airplane at 4000 feet altitude. Data from NACA 8-foot high-speed tunnel.

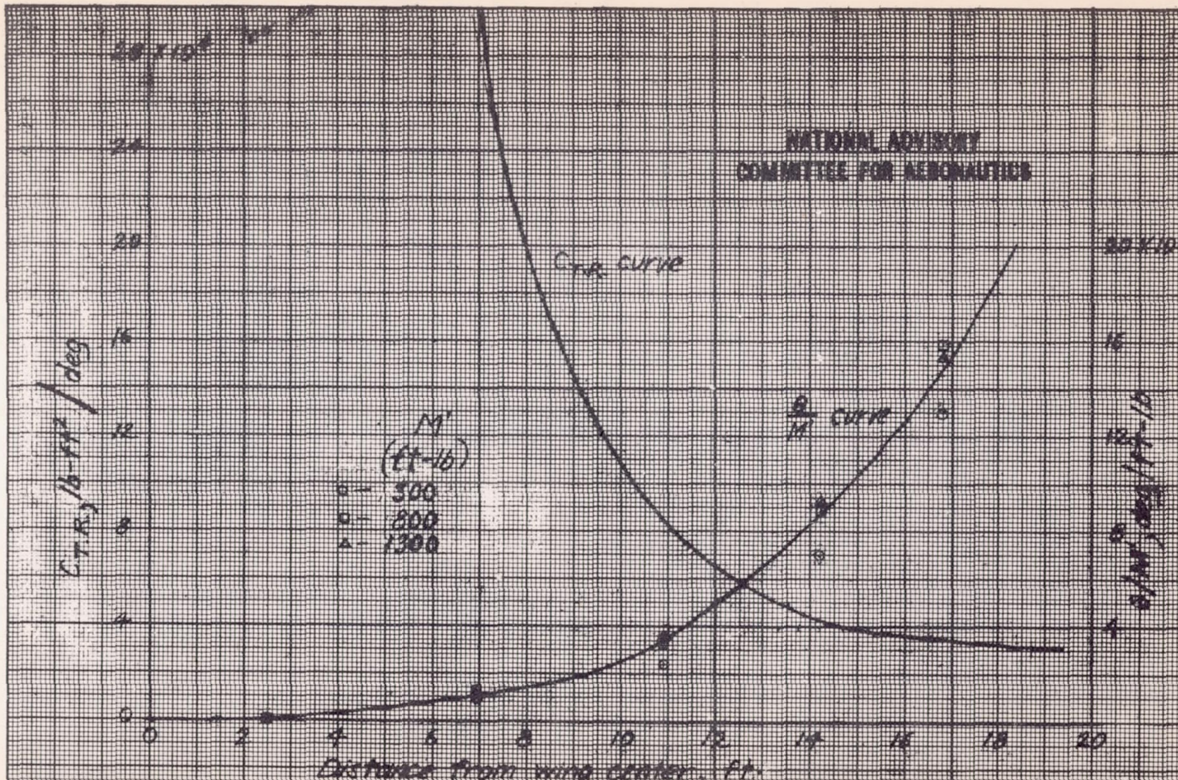

Figure 3.- Experimental data for spanwise distributions of twist and torsional rigidity for a concentrated torque applied at a section near the wing tip. P-47B airplane. Data from torsional rigidity tests made by Army Air Forces, Materiel Center, Wright Field.

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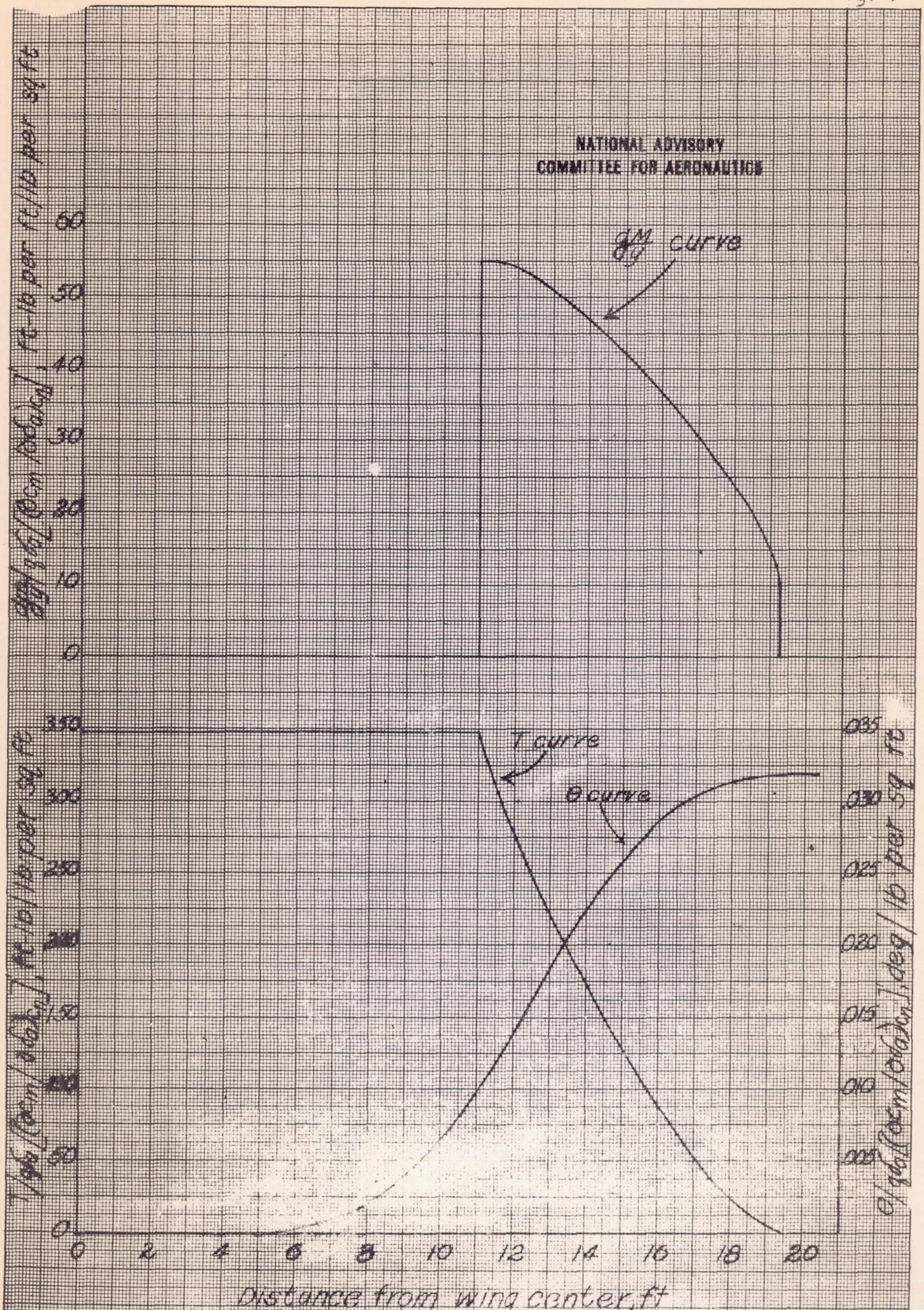

Figure 4 - Calculated distributions of dM/dy, T, and θ, resulting from a rolling maneuver for the P-47C-1-RE airplane.

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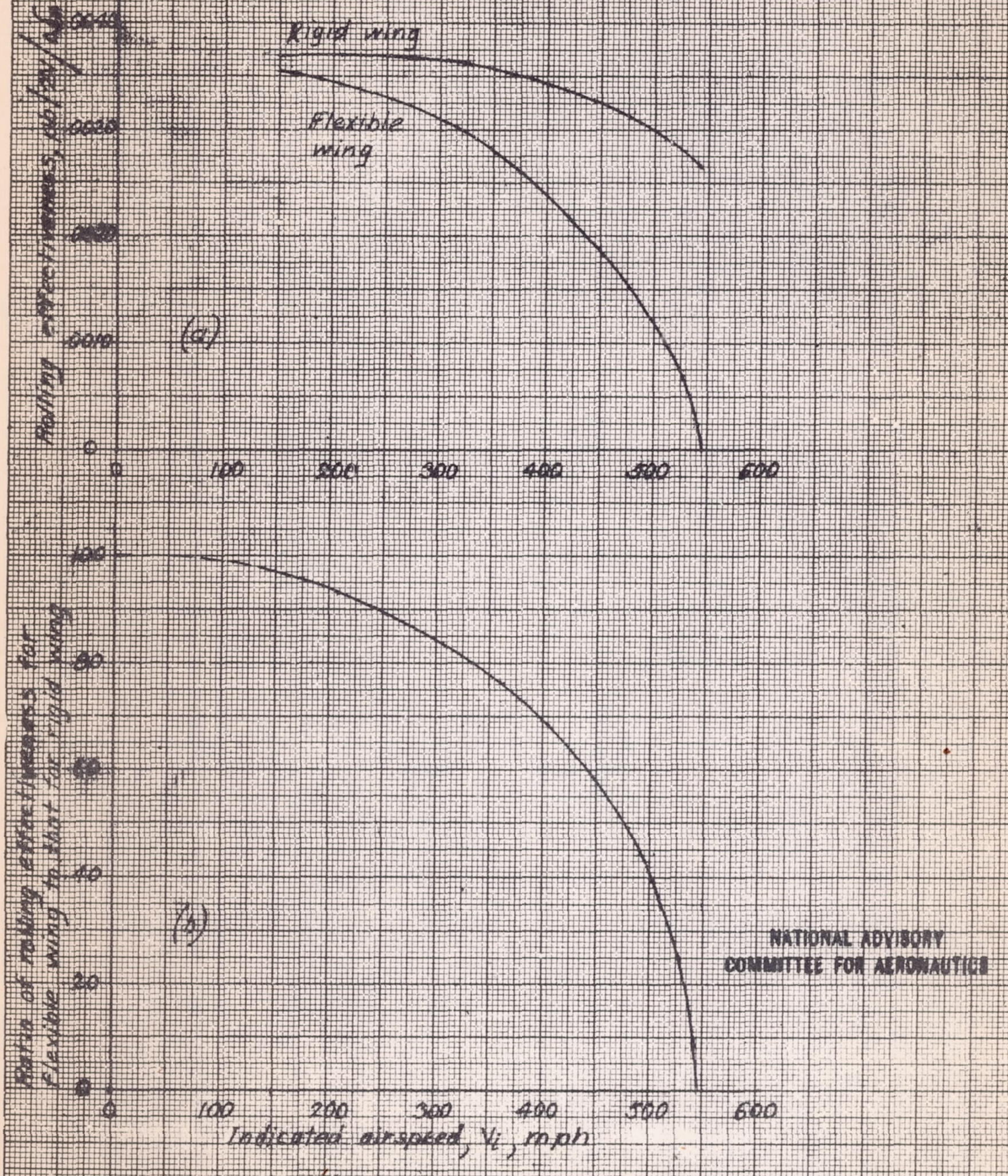

(a) Rolling criterion, $pb/2V/\Delta\delta_a$

(b) Ratio of rolling criterion for flexible wing to that for rigid wing

Figure 5.- Variation with indicated airspeed of calculated effect of wing flexibility on rolling. P-47C-1-RE airplane, $\Delta\delta_a = 8°$, altitude ≈ 4000 feet.

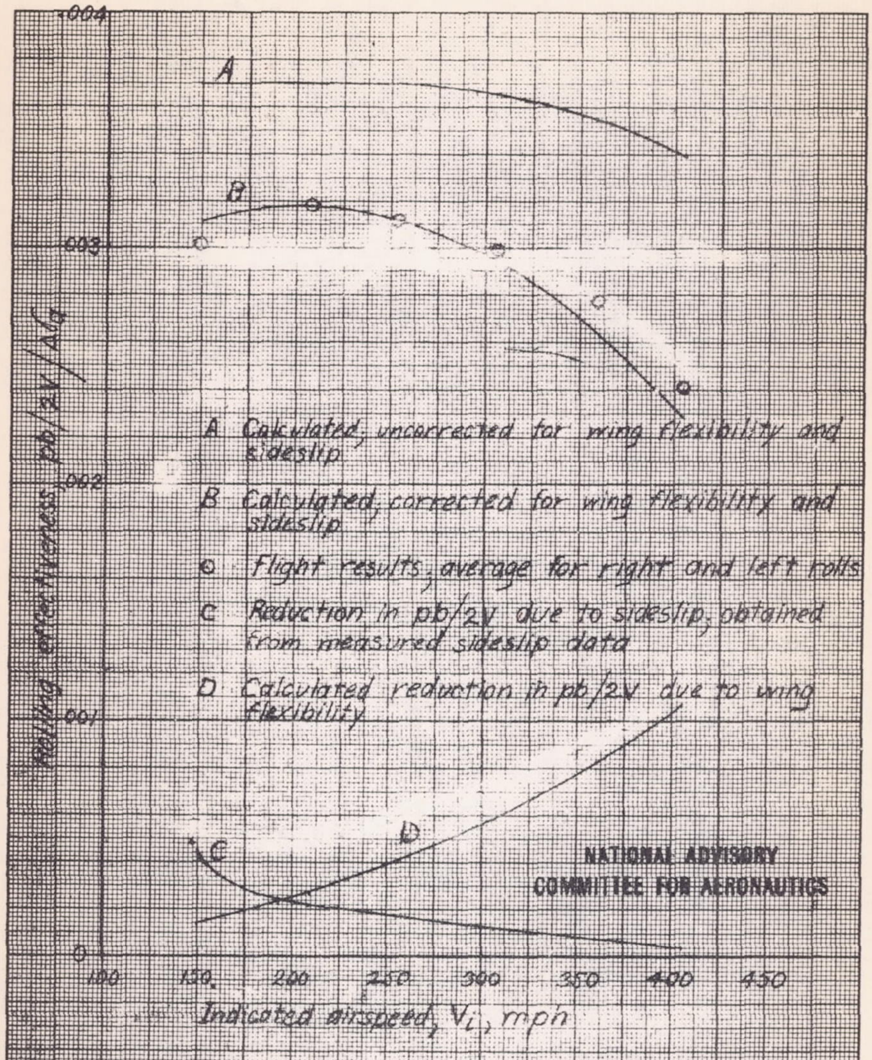

Figure 6.- Comparison of calculated rolling effectiveness with flight results for various indicated airspeeds. P-47C-1-RE airplane, $\Delta\delta_a = 8°$, altitude ≈ 4000 feet. Flight results from unpublished tests made at LMAL.